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A THREE-PARAMETER STOCHASTIC SUBMARINE TRAILING MODEL

William T. McCormick, Jr.
Alan W. Starr

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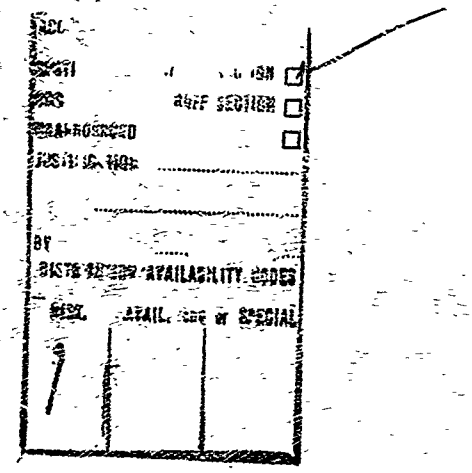

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A THREE-PARAMETER STOCHASTIC SUBMARINE TRAILING MODEL

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January 1971

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ABSTRACT

A method is presented for representing the performance of a trailing platform in maintaining trail against a submarine. The process of holding trail, loss, and unaided reacquisition is modeled as a three-parameter stochastic process. The parameters have direct physical interpretations and can readily be related to, or derived from, empirical exercise data. In addition, the model also includes a parameter that may be chosen to specify the length of time out of contact necessary to constitute a loss. This formulation allows the results of trailing experiments, which may be available in a variety of forms, to be used in predicting the effectiveness of trailing operations.

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I. INTRODUCTION

This paper considers one important aspect of the problem of trailing a submarine over an extended period of time. In general, the success of a trailing operation might depend on such factors as:

1. The ability of a remote surveillance system to detect, classify, and localize the submarine.
2. The ability of the trailing platform[1] to initiate trail.
3. The ability of the trailing platform to maintain trail, at least intermittently, without being counterdetected and with or without outside assistance.

The aspect considered here is the ability of the platform to maintain trail, and to reacquire trail after occasional losses of contact, without help from external sources. Although position information that might be provided by other platforms or by a remote surveillance system could be helpful in reacquiring contact, it is clear that any effective trailing platform must have some ability to recover from contact losses using only the information obtained during its most recent period of contact.

It is assumed here that trailing is accomplished by using acoustic sensors to detect noise radiated by the

[1]In this paper, "trailing platform" means either a single attack submarine, a single surface ship, or a number of aircraft operating such that one aircraft is always in the general vicinity of the submarine.

submarine. Since the trailed submarine might also be able to detect the trailer, the requirement that the trailer not be counterdetected would be likely to impose some constraints on the tactics used by the trailing platform. Unless the trailing platform has a very large detection advantage over the trailed submarine, these constraints would make the trailing operation an intermittent process, with frequent contact breaks ranging in duration from a few minutes to a permanent loss of contact. The cause of a contact break may be a change in the ambient noise, a change in the submarine's depth, or the local acoustic refraction properties of the ocean, or it may be due to one of many routine variations in the submarine's operating procedure. As time elapses since a loss of contact, regain of contact would become increasingly difficult, due to gradual growth of the location uncertainty area which must be searched. After a long period out of contact, the chance of a platform reacquiring trail without outside help would become small.

The representation of the "unaided" loss and reacquisition process described in this paper is the basis for a submodel that was used in a more general model for estimating the overall potential effectiveness of trailing operations. [1] The appendix describes how the submodel computes holding and reacquisition probabilities for use in this discrete-time trailing model.

II. GENERAL REPRESENTATION OF TRAILING PROCESS

As a result of oceanographic fluctuations and unknown tactical operating procedures, the history of contact breaks in a trailing operation may be thought of as a stochastic process. This and the following sections describe a procedure for representing trailing as a simple stochastic process.

It is assumed that trailing is conducted in a way which makes the probability of counterdetection by the trailed submarine negligible. This assumption means that the "random" tactical changes made by the trailed submarine will not depend on the actions of the trailer.

Furthermore, it is assumed that, during periods of contact, the instantaneous probability of losing contact is independent of how long contact has been held;[1] that is, the process of losing contact is assumed to be Poisson.[2] Thus, one may define a constant loss of contact frequency, λ_o, such that $\lambda_o\, dt$ is the probability that a contact loss of any duration will begin in an interval dt.

Similarly, during periods of loss of contact, one may define a reacquisition frequency $\mu(t)$, such that $\mu(t)\, dt$ is the probability of regaining contact in a time interval dt, given that contact has been lost for a time t. It is

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[1]This assumption neglects "learning" effects, which might cause trailing performance to improve somewhat during an extended period of contact.

[2]Only time <u>in</u> contact is included in the time variable in the process.

not possible, however, to assume that $\mu(t)$ is independent of t, because the reacquisition frequency falls off with time due to the increasing size of the position uncertainty area of the submarine. The exact functional form of $\mu(t)$ will depend on the particular reacquisition tactics employed by the trailer, and on the degree of randomness of the submarine's motion, but it will always be monotonically decreasing.

Now that the recontact frequency, $\mu(t)$, has been defined it is possible to derive an expression for $P_r(t)$, the probability that a platform will regain contact at least once in a time t since loss of contact. The probability of no regain up to time t' and a regain in the next interval dt' is

$$dP_r(t') = (1 - P_r(t'))\ \mu(t')dt' .$$

Rearranging terms and integrating gives

$$\int_0^t \frac{dP_r}{1-P_r} = \int_0^t \mu(t')dt'$$

or

$$1 - P_r = e^{-\int_0^t \mu(t')dt'}$$

and therefore

$$P_r(t) = 1 - e^{-\int_0^t \mu(t')dt'} .$$

Thus the probability that a lost contact will eventually be regained is unity only if

$$\int_0^\infty \mu\ (t')dt' = \infty .$$

If this integral is finite, then there is a finite probability that a loss of contact will be "permanent," unless recontact is provided by an external source.

III. GENERALIZATION OF THE DEFINITION OF THE TRAILING STATE

In the above discussion a submarine was considered to be under trail only when contact was currently being held, i.e., when an identified acoustic signal was being received by the trailing platform. The purpose of this section is to generalize the definition of the trailing state so that contact losses of short duration are not counted as periods out of trail. There are two motivations for this generalization:

- Brief losses of contact might be very numerous but would have little effect on the overall success of a trailing operation. In a complicated discrete-time trailing model, it is desirable to avoid expending a large computational effort on short-term multiple loss and regain events which would have almost no effect on the overall results.
- In compiling exercise data, a contact loss is usually defined to have some minimum duration; shorter breaks are ignored. In order to compare exercise results with the predictions of the model, or to use such empirical data as input to the model, the model must be compatible with various definitions of the trailing state.

Let τ be the minimum duration of a contact loss for it to be considered as a break in trail. Let $\lambda(\tau)dt$ be the probability that a contact break of duration τ or longer begins in the interval dt, given that contact was held at the beginning of the interval. Then

$$\lambda(\tau) = \lambda_0 \cdot \{\text{Probability that a loss of contact lasts for a time} \geq \tau\} \,.$$

When the quantity $1-P_r(\tau)$ is substituted for the probability in the braces,

$$\lambda(\tau) = \lambda_0 \, e^{-\int_0^\tau \mu(t')dt'} \,.$$

Note that the second factor in $\lambda(\tau)$ depends on the details of the reacquisition process. It should also be noted that it is not necessary to associate a time (such as τ) with a regain of contact, since any regain, no matter how short, is enough to provide the trailing platform with the latest information about the submarine's location.

The reciprocal of $\lambda(\tau)$ is the mean holding $T_H(\tau)$ until a loss of contact of duration τ or longer. In computing $T_H(\tau)$ from empirical data, all contact breaks shorter than τ would be counted as time <u>in</u> contact.

IV. SPECIFIC 3-PARAMETER REPRESENTATION

In order to continue the analysis of the loss and regain process, it is necessary to specify a functional form for the reacquisition frequency $\mu(t)$. For very short contact breaks, which may occur frequently, the trailing platform would be expected to take no special action, but merely wait for the acoustic signal to reappear. The growth of the uncertainty area during these short breaks may be neglected, and a constant[1] recontact frequency μ_o may be assumed. However, the recontact frequency would gradually decrease for larger time, t, since loss of contact, because it is assumed that the trailing platform would commence a search procedure involving sweeping out area at a constant rate within a constantly expanding area[2] of uncertainty of the target.

The rate at which $\mu(t)$ decreases with t depends on many environmental and tactical details. However, it is reasonable to assume that the shape of the area of uncertainty remains approximately constant, while its area increases with the square of time. This assumption and the assumption of a constant search rate leads to the following functional form for $\mu(t)$:

$$\mu(t) = \frac{\mu_o}{1 + \gamma^2 t^2} .$$

---

[1]This means that the short-term regain process is assumed to be Poisson.

[2]The uncertainty area would be expected to grow roughly as t^2.

Note that $1/\gamma$ is the time required for the recontact frequency to decrease by half due to expansion of the area to be searched. This time is clearly longer if either the searching platform's search rate is high or if the trailed submarine's change in velocity is small.

If this expression for $\mu(t)$ is inserted in equation (1) for $P_r(t)$ then the probability that a platform will regain contact at least once in a time t since loss is:

$$P_r(t) = 1 - e^{-\frac{\mu_o}{\gamma^2}\int_o^t \frac{1}{1/\gamma^2 + t^2}\,dt}$$

$$= 1 - e^{-\frac{\mu_o}{\gamma}\tan^{-1}\gamma t} \quad .$$

The unconditional probability of regaining contact is then

$$P_r(\infty) = 1 - e^{-\frac{\pi}{2}\frac{\mu_o}{\gamma}} \quad .$$

Recalling that the calculation of $\lambda(t)$ involved the reacquisition process, it may now be computed:

$$\lambda(\tau) = \lambda_o \cdot \text{probability}\{\text{a loss of contact lasts for a time} \geq \tau\}$$

$$= \lambda_o\, e^{-\frac{\mu_o}{\lambda_p}\tan^{-1}\gamma\tau} \quad .$$

The mean holding time until a loss lasting time τ or longer is then:

$$T_H(\tau) = \frac{1}{\lambda_o}\, e^{\frac{\mu_o}{\lambda_o}\tan^{-1}\gamma\tau} \quad .$$

It should be ncted that Dobbie [2], [3] in two papers on ASW detection and tracking operations, used the following functional form for $\mu(t)$:

$$\mu(t) = \frac{\mu_o}{1 + \frac{c}{\mu_o+c}\left(e^{(\mu_o+c)t}-1\right)} \cdot$$

For this function the probability that a platform will regain contact at least once in time t is

$$P_r(t) = \frac{\mu_o}{\mu_o+c}\left(1-e^{-(\mu_o+c)t}\right),$$

and the unconditional probability of regaining contact is

$$P_r(\infty) = \frac{\mu_o}{\mu_o+c} \cdot$$

The mean holding time until a loss lasting time τ or longer is then

$$T_H(\tau) = \frac{1}{\lambda_o} \cdot \frac{1}{1 + \frac{\mu_o}{\mu_o+c}\left(e^{-(\mu_o+c)\tau}-1\right)} \cdot$$

Dobbie chose this specific form because it was suitable for an approach using Laplace transform techniques. However, in the more complex model [1] in which the present analysis has been embedded, Laplace transform methods are not helpful, and the functional form for $\mu(t)$ was therefore chosen to have a simple physical interpretation of the "position uncertainty spreading parameter," γ, and also to give a good fit to exercise data. A comparison of the two functions is given in Fig. 1, where the parameters have been chosen so that the probabilities of a permanent loss of contact are equal.

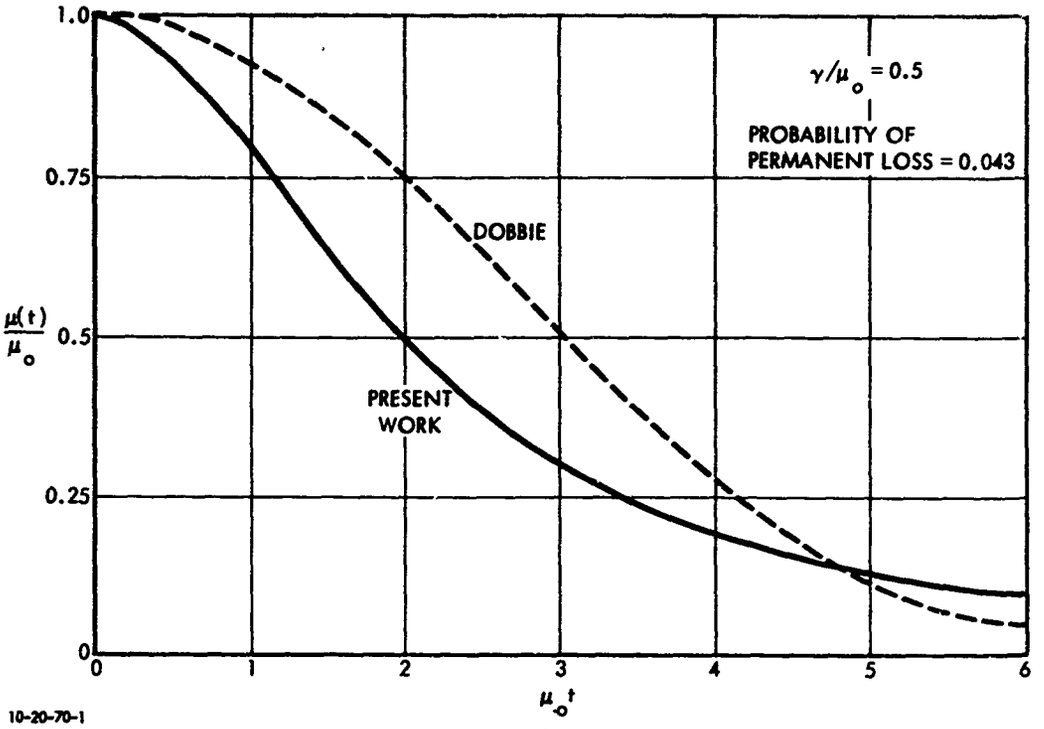

FIGURE 1. Comparison of Two Alternative Functional Forms for Reacquisition Frequency

V. DETERMINATION OF TRAILING PARAMETERS FROM EXERCISE DATA

The three parameters representing the trailing process could be derived from more detailed tactical models, or they could be generated empirically. In either case, it is useful to be able to relate the parameters to data from actual trailing exercises. It is assumed here that such data would be available in the form of a record of all times when contact was lost or regained. Since it is possible that some or all of the very short term losses may not have been recorded, it is important that the data be used in a way which is not sensitive to these omissions.

For the regain frequency function chosen in the previous section, the mean holding time until a loss of contact of duration τ or greater was shown to be

$$T_H(\tau) = \frac{1}{\lambda_o} e^{\frac{\mu_o}{\gamma} \tan^{-1} \gamma \tau} .$$

The approach taken here is to construct an empirical version of this function, $T_H(\tau)_{emp}$, directly from the exercise data, and then choose the three parameters μ_o, λ_o, and γ to fit this curve. Analyses of exercise data often define a loss of contact as being out of contact for some minimum time τ, making it possible to ignore very short-term losses of contact which may occur frequently but which may have little effect on trailing capabilities. Unfortunately, different

analyses often use different values of τ to define their losses, so that results are hard to compare. One advantage of the present procedure is that it produces $T_H(\tau)$ for all τ, thus allowing comparisons to be made with all available exercise analyses, regardless of how a loss of contact was defined.

The empirical function $T_H(\tau)_{emp}$ is computed as follows:

- Select a small value of τ (say, 0.1 hours).
- Let $N(\tau)$ be the total number of contact periods ignoring contact breaks of duration less than τ.
- Let $H(\tau)$ be the corresponding total number of hours in contact, again not counting losses less than τ in length.
- Then the observed mean time till loss of contact of duration τ or greater is computed as follows:

$$T_H(\tau)_{emp} = \frac{H(\tau)}{N(\tau)+1} \quad .$$

Successively larger values of τ may then be chosen and the entire procedure can be repeated to generate the function $T_H(\tau)_{emp}$.

The parameters λ_o, μ_o, and γ are then obtained as follows:

- At $\tau=0$, the model predicts that $T_H(0) = \frac{1}{\lambda_o}$, and thus λ_o is found by taking the inverse of the intercept of $T_H(\tau)_{emp}$.
- For small τ, $T_H(\mu)$ goes as $\frac{1}{\lambda_o} e^{\mu_o \tau}$ and therefore when plotted on a logarithmic scale the slope of $T_H(\tau)_{emp}$ near $\tau = 0$ gives μ_o.

- The final free parameter, γ, is then selected to produce the "best" fit (in some appropriate sense) between $T_H(\tau)$ and $T_H(\tau)_{emp}$ over the time period of interest.

Figure 2 shows the result of applying this procedure to simulated exercise data. The quality of the fit shown in Fig. 2 is comparable to the fits obtained using real exercise data. The straight dashed line indicates that the simpler Poisson model for reacquisition ($\mu(t)$=constant) gives an unsatisfactory fit. Clearly, the effect of spreading of the position uncertainty area cannot be omitted from the trailing model. For comparison, the fit given by Dobbie's model is shown for two values of his parameter, c. Although Dobbie's model would probably be acceptable for many applications, all available exercise data was more closely approximated by the reacquisition model presented in this paper.

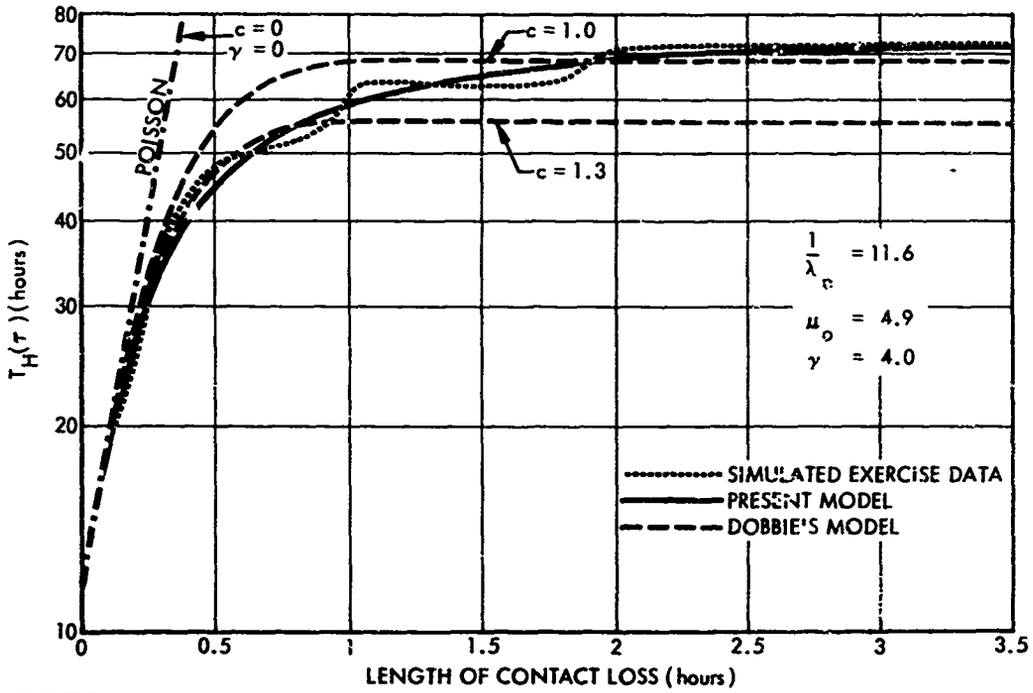

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FIGURE 2. Comparison of Mean Holding Time, as a Function of Time Used to Define Minimum Loss of Contact, for Present Model, Dobbie's Model, and Simulated Exercise Data

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APPENDIX

HOLDING AND REACQUISITION PROBABILITIES

HOLDING AND REACQUISITION PROBABILITIES

The process of trailing, loss, and reacquisition has been represented above as a 3-parameter stochastic process. This representation could be used in a simple continuous trailing model, where the evolution of the fraction of submarines under trail is governed by a differential equation. However, many large-scale models, because of their complexities and interrelated submodels, must be formulated in a discrete time framework. This appendix discusses how the trailing formulation described above was implemented in the nonstationary multistage Markov model described in Ref. 1.

A multistage model requires various holding and reacquisition probabilities (which relate to the interval length T) such as

- The probability that trail is held at stage k + 1, given that it was held at stage k (denoted $P_H(T)$);
- The probability that trail is held at stage k + 1, given that it was not held at stage k but was held at stage k - 1 (denoted P_{reacq});
- The probability that trail is held at stage k + 1, given that it was not held at stages k or k - 1 but was held at stage k - 2 (denoted P'_{reacq});

etc. The different numbers of time intervals that the submarine has been out of contact are represented by different "lost states" in the Markov model.

The choice of the time interval length T in the multistage model depends on several conflicting considerations.

It should be chosen short enough so that large numbers of events (i.e., losses and regains of contact, etc.) do not occur in a single interval, and so that various environmental parameters may be considered constant during the interval. On the other hand, it should be chosen long enough so that the values of certain other random variables (not those represented in the trailing model) at two successive stages may be considered uncorrelated, and so that an excessive number of "lost" states need not be defined to adequately represent the trailing history of the submarine.

All of these requirements on the choice of T can be satisfied by taking advantage of the feature of the stochastic formulation described above in Section 3. This feature allowed the state of being in trail to be defined such that all contact breaks of duration τ or less could be ignored. As long as

$$T < 2\tau ,$$

multiple contact losses cannot by definition occur in a single interval. Thus, T can first be chosen large enough to satisfy the various other requirements, and the complication caused by large numbers of relatively insignificant contact breaks in a single interval can then be avoided by choosing $\tau = T/2$.

A. CALCULATION OF HOLDING PROBABILITIES

The first probability of interest is $P_H(T)$, the probability that a platform is holding contact at the end of a time interval of length T, given that it held contact at the beginning of that interval. This probability is made up of the sum of the probabilities of two exclusive

events: contact is never lost throughout the interval, or contact is lost during the interval and then regained before the end of the interval. Therefore, $P_H(T)$ is expressed as follows:

$$P_H(T) = e^{-\lambda(\tau)T} + \int_0^{T-\tau} e^{-\lambda(\tau)t} \lambda(\tau) dt \int_{t+\tau}^{T} \left[1-P_{R\tau}(t'-t)\right] \mu(t'-t) e^{-\lambda(\tau)(T-t')} dt'$$

where

$\lambda(\tau)$ = the platform loss-of-contact frequency for a time τ or greater,

$\mu(t)$ = the platform reacquisition frequency at time τ,

and $P_{R\tau}(t)$ is the probability of at least one unaided reacquisition in a time $t > \tau$, given that no reacquisition occurred before time τ, and is calculated as follows:

$$P_{R\tau}(t) = 1 - \frac{1-P_R(t)}{1-P_R(\tau)} \text{ for } t>\tau, \text{ otherwise } P_{R\tau}(t) = 0$$

where $P_R(t)$, the probability of at least one recontact in time t after loss, is identical to the probability $P_r(t)$ derived in the main text. In the integrals in the second term above, t represents the time (during the interval of length T) at which the loss lasting τ or longer occurred, and t' represents the time at which contact was regained and held for the remainder of the interval.

B. CALCULATION OF REACQUISITION PROBABILITIES

1. Out of Contact One Interval

P_{reacq} is defined as the probability that a platform would be in contact at stage k + 1, given that it was out

of contact (in the lost state) at stage k and in contact at stage k - 1, and given that no help was obtained from another platform. This probability can be calculated by merely considering the two different ways in which the platform can progress from being in contact at interval k - 1 to being in contact at interval k + 1.

The exact probability that the platform is holding two intervals later is therefore the sum of the probabilities of the two exclusive events of (being in contact at the end of the first interval and also being in contact at the end of the second interval) and (not being in contact at the end of the first interval but being in contact at the end of the second). Therefore,

$$P_{exact}(2T) = P^2_H(T) + [1 - P_H(T)]P_{reacq} .$$

$P_{exact}(2T)$ can be calculated approximately, using the functions $P_H(t)$ (derived in the previous section) and $e^{-\lambda t}$ evaluated at $t = 2T$. Since the calculation of $P_H(2T)$ only includes the possibility of one event of a loss and regain while 2T is a sufficient time for three such events to occur, $P_{exact}(2T)$ must also include correction terms that account for more than one event. The probability P_Δ of the event of a loss and a reacquisition in an interval of length T is just

$$P_\Delta = P_H(T) - e^{-\lambda(\tau)T} .$$

It may be verified that as long as P_Δ is small, the probability of two such events occurring in two intervals of length T is approximately $2P^2_\Delta$, and the probability of three such events in two intervals is proportional to P^3_Δ. Therefore,

$$P_{exact}(2T) = P_H(2T) + 2P_\Delta^2 + O(P_\Delta^3)$$

or for small P_Δ,

$$P_{exact}(2T) \approx P_H(2T) + 2P_\Delta^2 .$$

Substituting this expression for $P_{exact}(2T)$ into the above expressions involving P_{reacq} and solving for P_{reacq}, we find that

$$P_{reacq} \approx \frac{P_H(2T)+P_H^2(T)-4P_H(T)e^{-\lambda(\tau)T}+2e^{-\lambda(\tau)T}}{1 - P_H(T)} .$$

2. Out of Contact Two Intervals

If the definition of the states includes a "lost for two intervals state" which can only be entered from one of the other lost states, then the model requires the calculation of the probability of reacquisition from this state, P'_{reacq}.

P'_{reacq} is defined as the probability that a platform will be in contact at the beginning of the interval beginning at stage k + 1, given it was out of contact both at stages k and k - 1, but in contact at stage k - 2. This quantity is calculated in a manner similar to that which was used to calculate P_{reacq}, but in addition to P_H the calculation also includes P_{reacq}. The exact probability of being in contact three intervals later, given that the platform was in contact, is

$$P_{exact}(3T) = P_H^3(T) + 2\cdot P_H(T)\cdot\left[1 - P_H(T)\right]P_{reacq} + \left[1 - P_H(T)\right]\left[1 - P_{reacq}\right]P'_{reacq} .$$

It can be shown that for small P_Δ

$$P_{exact}(3T) \approx P_H^3(3T) + 8P_\Delta^2 \ .$$

When this expression is substituted in the equation containing P'_{reacq} and solved for P'_{reacq}, we find

$$P'_{reacq} = \frac{\left[P_H^3(3T) + 8\left(P_H - e^{-\lambda(\tau)T}\right)^2 + P_H^3(T) + 2P_H(T)\cdot\left[1 - P_H(T)\right]P_{reacq}\right]}{\left[1 - P_H(T)\right]\left[1 - P_{reacq}\right]P_{reacq}} \ .$$

If necessary, more lost states could be defined and the appropriate reacquisition probabilities from these lost-from-contact states could be computed approximately in a manner similar to that shown above.

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13 ABSTRACT

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